PX 266: Geophysics

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Disclaimer:
This revision guide is not intended to be used instead of going to lectures and is only supposed to summarise the lecture course and aide revision. Despite our hardest work it is possible that there may be mistakes in this guide so remember that revising from your own notes will also help and prevent any problems.

1 - The Basic Characteristics of the Earth

We can assume (to first approximation) that the Earth is a sphere (compare the heights/depths of mountains/trenches ~ 10 km to the radius of the Earth ~ **6,400 km**). A better approximation is that of an oblate spheroid because the Earth bulges out around the equator, modelled by the equation:

$$R_E(\lambda) = R_{Equator}\left(1 - f\sin(\lambda)^2\right)$$

Where $f \approx \frac{1}{300}$, this leads to a difference in radius from the pole to the equator of about 22 km.

It is possible to measure the mass of the Earth by measuring the orbital period (T) of a satellite of known mass (m) orbiting in a circular orbit at radius r:

$$\frac{G M_{Earth} m}{r^2} = \frac{(2\pi)^2 mr}{T^2}$$

which comes from balancing the gravitational and centripetal forces. This gives the Earth's mass as: **5.97 x 10^{24} kg**.

The Earth's moment of inertia can be inferred from the precession of its orbital axis (the precession of the equinox)

$$I_{Earth} = 0.331 \mathrm{M}_E R_E^2$$

c.f. a uniform sphere:

$$I = 0.4\mathrm{MR}^2$$

The coefficient being smaller in the case of the Earth tells us that it is denser towards the centre.

The age of the Earth is generally accepted to be **4.54 Ga** (giga-annum, i.e. billion years), we reach this number from (e.g.) radiometric dating of meteorite sample.

2 - The internal structure of the Earth

Most of the evidence for the internal structure of the Earth comes from seismology, it gives us the following picture (from Lecture 2 of Dr. Bell's course)

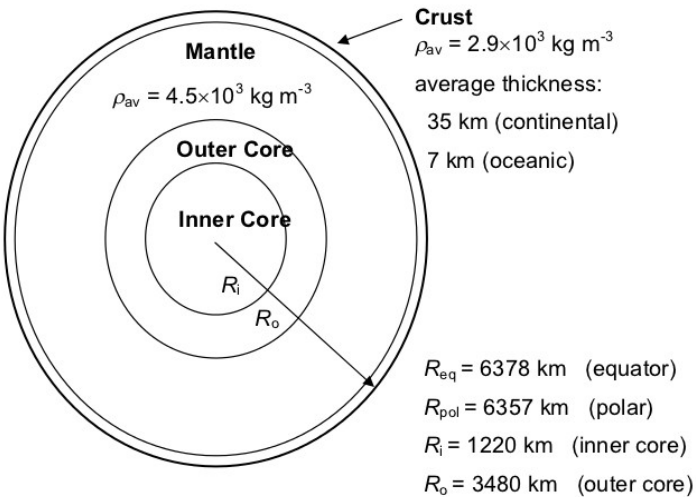


Figure 1: The "onion Earth" [1]

There are two different types of crust *Continental* and *Oceanic* (50/7 km thick, silicic crystalline/basalt, low density/denser than continental crust, respectively).
The mantle is denser and hotter than the crust, it is less rich in silica and is mainly basalt with some (Mg, Fe) silicate minerals (note that there are changes in the crystal structure of minerals at circa 400 and 700 km depths).

The core is mostly iron and fairly uniform. the outer core is liquid while the inner is a "pressure frozen" solid. This is electrically conducting and is the "geodynamo" the generates the Earth's magnetic field (see section 8)

We can also talk about the mechanical as well as chemical properties of layers e.g. liquid/solid? How does it respond to stresses (forces)? (is it an elastic solid? A viscous fluid?). This gives several quantities of interest: The density ρ, viscosity η, and the elastic moduli (bulk k and shear μ).

Using mechanical definitions we can talk about the *Lithosphere* and the *Asthenosphere*. The Lithosphere encompasses the whole crust and some of the upper mantle (it reaches a depth of about 200 km). It is a rigid, elastic solid but under high stress fractures brittlely. While the Asthenosphere behaves as a viscous fluid over LONG (geological) timescales but behaves as an elastic solid on short timescales.

Note that as depth increases: density, elastic constants, pressure, temperature and mantle viscosity increase!

3 - The formation of the Earth and the Solar System.

Around the sun a dust disk gathered and the following happened:

1. Coalescence of "planetesimals" ($\leqslant$ 1 km across).
2. Planetesimal growth up to ~ 1000 km
3. Collisions between planetesimals cause ACCRETION of larger bodies (moon, mars, earth sized).

According to computer models, stages 1&2 should take ~ 10^5 years while stage 3 takes about 10^7-10^8 years. (Note that during stage 3 large scale collisions are possible which could explain the Earth's axis tilt).

As the Earth developed the energy sources were impact KE, gravitational PE (from compression) and radioactive decay. The heating of the early Earth leads to the formation of the layered structure, e.g. the "iron catastrophe", as the Earth heats, iron melts and flows towards the centre of the Earth, more heating means more melting etc.

Radiometric dating of Chondrite meteorites (most common type of meteorite) shows that the solar system is approximately 4.5-4.6 Ga old (they haven't been subjected to any geological processes and so are considered "typical" early solar system material). Compare this to the oldest minerals found on Earth which are approximately 4.3 Ga old (in Australia). Note that moon rocks also dte to approximately 4.5 Ga.

For radiometric dating recall that:

$$T_{1/2}=\frac{ln2}{\lambda}$$

Which is the half life is equal to the natural logarithm of 2 divided by the decay constant.
We can use a "Radiometric clock" if we know how many daughter (D(t)) nuclei there are compared to the number of parent (N(t)) nuclei.

$$t=\frac{1}{\lambda}\ln\left[1+\frac{D(t)}{N(t)}\right] \quad \text{(X)}$$

Assume that the number of daughter nuclei is 0 at t=0 (some time in the geological past), also note that this only applies for stable daughter nuclei.

4 - Method of Isochrons

It would obviously be a challenge to measure D(t), N(t) individually instead we measure isotopic ratios using a mass spectrometer.

If we modify equation X to be, where we do not assume D(0)=0:

$$D(t)=N(t)\left[e^{\lambda t}-1\right]+D(0)$$

Then, choosing a stable, non-radiogenic isotope (R) which is the same chemical element as D we can normalise the equation w.r.t. R(t):

$$\frac{D(t)}{R(t)}=\frac{N(t)}{R(t)}m+\frac{D(0)}{R(0)}$$

Where

$$m=e^{\lambda t}-1 \quad \text{(Y)}$$

Also R(t)= R(0) as R is stable.

Using this relation, take several different mineral samples from a rock and plot $\frac{D(t)}{R(t)}=y$ against $\frac{N(t)}{R(t)}=x$, the result should be a straight line with gradient m, so can find t using equation Y.
This 't' is the last time the rock was open to the exchange of nuclei by non-radioactive means (such as melting, chemical reation etc.) so the amount of scatter away from a straight line indicates how good "closure" the system has had over the rock's history. Provided there are enough data points whole-rock isochron is "self checking" for system closure (more points gives a better fit of the line). There are a whole variety of radioactive materials that are useful for different time periods due to varying half lives.

5 - Gravity.

For point masses Newton's law of gravity applies:

$$\underline{F}=\frac{Gm_1 m_2}{|\underline{r}_1-\underline{r}_2|^2}\frac{(\underline{r}_1-\underline{r}_2)}{|\underline{r}_1-\underline{r}_2|}$$

where $\underline{r}_1$ and $\underline{r}_2$ are the vector locations of masses m_1 and m_2 (respectively) from a common origin. However, in the case of an extended body it is necessary to take an integral over the volume of the body:

$$\underline{F}(\underline{r})=\int\int\int Gm\rho(\underline{r}')\frac{(\underline{r}'-\underline{r})}{|\underline{r}'-\underline{r}|^3}d\underline{r}$$

Note that it is also possible to calculate potential:

$$\underline{V}(\underline{r})=\int\int\int -\frac{G\rho(\underline{r}')}{|\underline{r}'-\underline{r}|^3}d\underline{r}$$

It is important to note that the gravitational behaviour of spherical shells is that of a point mass at their centre (and the Earth is simply layers of "spherical" shells). Inside a shell there is no field. Gravity only arises from a mass at a smaller radius than the measuring point.

If the Earth were uniform spherical shells, would expect a spherical gravitational potential, i.e. constant potential at constant height. However, differences in measured g from expected g ("gravity anomalies") are found, these tell of a different density structure underneath the surface of the Earth to the one expected.

When calculating the Earth's gravity, it is simple to build the picture up in stages:

1. Spherical, non-rotating Earth: $V=\frac{-Gm_E}{r}$
2. Rotating spherical Earth: Have centripetal acceleration to account for, get a new potential U: $U=V+\frac{\omega^2 r^2\sin^2(\theta)}{2}$ where θ is the spherical polar angle from the North to South pole.
3. Adjust for the Earth being an oblate spheroid (take account of variations in R_E).

This gives the reference gravity formula:

$$g(\lambda)=g(0)\left(1+\delta\sin^2(\lambda)+\epsilon\sin^4(\lambda)\right)$$

where $\lambda=90°-\theta$, $\delta=5.279\times10^{-3}$, $\epsilon=2.346\times10^{-5}$ and $g(0)=9.780\,ms^{-2}$ at the equator.

The sea level equipotential is called the *Reference Spheroid*, the actual sea level equipotential surface is called the *geoid* within ±100m of the reference spheroid.

Satellite data can measure disturbances to within ±1 mgal (field) or ± 2 cm (potential). Land based measurements have sensitivity from ± 1 mgal to $\pm10^{-2}$ mgal.

Near mountains, plum lines are expected to be deflected, but the measured deflection is smaller than calculated, there are two main theories behind this:

1. The *Pratt hypothesis* suggests that mountainous material is less dense than other material and floats on the asthenosphere (also suggests that oceanic crust is more dense).

2. The *Airy hypothesis* predicts that the lithosphere density remains constant and there are mountain roots (observed) and oceanic crust is thinner (also observed).

The reality of isostatic compensation is a combination of the two models.

Examples of isostatic compensation are: Mountain ranges, the Dead Sea basin, Cross-USA Bouger anomaly and ice sheets.

6 - Seismology

Seismology is the study of elastic waves travelling through the Earth or on the Earth's surface. When considering isotropic elastic media, only two elastic constants are needed, use either:

- Young's modulus E with Poisson ratio υ.
- Bulk modulus K with shear modulus μ.

If hydrostatic pressure is applied to an elastic medium we define the bulk modulus such that:

$$k=-\frac{dp}{(dV/V)}>0$$

The shear modulus can be defined by putting shear stress on a medium:
to first order there is no volume change. A rough definition of μ is that it describes the amount of shape distortion under shear stress. It is possible to derive two wave equations:

1. For compressional waves:
$$\frac{\partial^2\phi}{\partial t^2}=\alpha^2\nabla^2\phi$$
$\phi=$scalar displacement potential (describes volume change)
$$\alpha=\sqrt{\frac{k+\frac{4}{3}\mu}{\rho}}\text{ is the wave speed, }\rho\text{=density}$$
2. For shear waves:
$$\frac{\partial \underline{\Psi}}{\partial t^2}=\beta^2\nabla^2\underline{\Psi}$$
$\underline{\Psi}=$vector displacement potential (describes shape distortions)
$$\beta=\sqrt{\frac{\mu}{\rho}}\text{ is the wave speed}$$

There are four different types of seismic wave:

$$\left\{\begin{matrix} P-wave \\ S-wave \end{matrix}\right\}\text{In bulk}$$
$$\left\{\begin{matrix} Love\,wave \\ Rayleigh\,wave \end{matrix}\right\}\text{on surface}$$

Bulk waves emanating from a point source are spherical so the energy density drops as $1/r^2$ while surface waves have a circular wavefront and the energy density drops as 1/r. (Note also that the energy density is proportional to the square of the wave amplitude).

As k, μ are always positive, α> β (so P-waves are always faster than S-waves).

Seismic waves can be affected by the processes of reflection, refraction and mode conversion (i.e. changing from P to S).
Body waves can be reflected at an interface (e.g. surface or core-mantel boundary). Moving into the Earth's interior the wave speed generally increases, paths can be described by the principle of least action of by Snell's laws.

Note that as the outer core is liquid and μ=0 for liquids there are no shear waves through the outer core. (For more information see the "Seismology" handout).

It is possible to locate earthquakes using seismometers. Measuring the time interval (Δt) between the arrival of the P-waves and the arrival of the S-waves gives:

$$\Delta t=\frac{x}{\beta}-\frac{x}{\alpha}$$

where x is the distance from the focus (origin of the earthquake) to the seismometer. Using three or more seismometers, the location of the focus can be accurately pinpointed.

It is possible to "invert" the travel times and get $\alpha(r)$ and $\beta(r)$ as a function of depth.

It is also possible to derive $\rho(r), k(r), \mu(r)$ from $\alpha(r), \beta(r)$ using the Adams-Williamson equation (as derived in lectures):

$$\frac{d\rho}{dr}=\frac{-GMr\rho(r)}{r^2\left[\alpha^2-\frac{4}{3}\beta^2\right]}$$

The derivation occurs when this is integrated numerically from the Earth's surface inwards (in steps of Δr).

Note that for practicality (as the crust is inhomogeneous), tend to start at the crust-mantle boundary. Also, as discontinuities are handled badly, the calculation is normally reset at the core-mantle boundary but this requires extra constraints! These are:

- At r=0, m_r=0 (no mass right at the centre of the Earth).
- Moment of inertia of the Earth
- Whole Earth oscillation modes

These can be used to constrain $\rho(r), k(r)$ and $\mu(r)$

There are problems with the A-W equation (as it travels deeper into the Earth), including that it ignores the effects of temperature distribution.

Seismic Tomography builds up a 3D picture of variation in α and β (up to now radial variation is all that was considered). (An example is the ability to confer temperature variation in the mantle and relate this to plate tectonic activity and "convection" in the mantle).

Earthquakes result from the sudden release of strain energy in the Lithosphere. Brittle fracture occurs along the fault line. There are 4 types: thrust fault (two plates pushing together), normal fault (they slide apart), strike slip (horizontal movement), oblique fault (horizontal slide and slide apart).

The size of an earthquake quantified by logarithmic *intensity* and *magnitude* scales. The intensity scale is qualitative (based on observation), useful for comparing to historical records. The magnitude scale is quantitative (based on measurements of seismic waves or seismic movement).

Consider a simple harmonic seismic wave:

$$A=A_{max}\cos(kx-\omega t)$$

The kinetic energy density is:

$$= \frac{1}{2} \rho \left(\frac{dA}{dt} \right)^2$$

$$= \frac{1}{2} \rho A_{max}^2 \omega^2 \sin^2 (kx - \omega t)$$

So the energy released in the quake is proportional to $(A_{max} \omega)^2$

It is possible to measure both A_{max} and ω from a seismogram
So a general form for magnitude M is:

$$M = \log_{10} (A_{max} \omega) + \text{correction}$$

The main correction is to compensate for the distance from the source to the seismometer. The energy density drops due to spreading of the wavefront and attenuation.

7 - Plate tectonics

In 1596, Ortelius noted that the African and American continents fit together like a jigsaw, there is also geological evidence that continents have moved. Plate tectonics explains continental drift.

Most geological activity occurs at plate boundaries, of which there are three types:

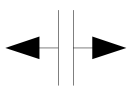
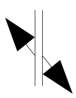
1. Constructive (new plate material created, normally mid ocean ridge)
2. Destructive (plate material destroyed, normally oceanic crust is "subducted" below continental crust)
3. Conservative (plates slide past each other, nothing is created or destroyed)

There is notation to describe this:

$$\left| {}_B\overline{V}_A \right| = \left| {}_A\overline{V}_B \right| = V_0$$

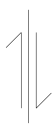
i.e. the velocity of A relative to B is equal to the velocity of B relative to A.

For constructive boundaries we can have normal or oblique spreading, denoted respectively:

At a destructive boundary the following symbol is used (pointing away from the plate being subducted):

At a conservative boundary:



When talking about plate velocities we nearly always deal with relative velocities (as there is no fixed point to measure against). The magnitude of velocities are typically mm to cm/year. Where 3 plates meet, relative velocities can be added, e.g. ${}_C\overline{V}_B = {}_C\overline{V}_A + {}_A\overline{V}_B$. For large plates, cannot assume a "flat Earth", need to use rotation poles.

The problem is that the relative velocity changes along the plate boundary on a spherical Earth. Using Euler's Fixed Point Theorem applied to tectonic plates we say that any plate motion is equivalent to rotation about an axis through the centre of the Earth. An instantaneous plate velocity can be described by a rotation speed and the relative rotation axis.
The axis will intersect the Earth's surface at 2 points (just a latitude and longitude, does not necessarily refer to any specific geographical feature).
Each pair of plates in contact has an associated rotation pole and angular speed:

$$V = \omega R_E \sin(\Delta)$$

Where ω is the angular velocity and Δ is the angular distance from the positive rotation pole (P) and the point of interest where the relative velocity is being obtained. (Note that it is irrelevant which is the positive and negative pole as long as they are kept consistent).

On a small circle about P, the velocity is constant, and if a boundary lies on this circle it is a conservative boundary.

Measuring part plate motions is possible with the evidence of paleomagnetism (measure magnetic inclination - tells of the latitude at which a mineral was magnetised - and the polarity).

8 - Earth's Magnetic Field

To first approximation, the Earth's magnetic field is just like that of a giant bar magnet. (Dipole field).

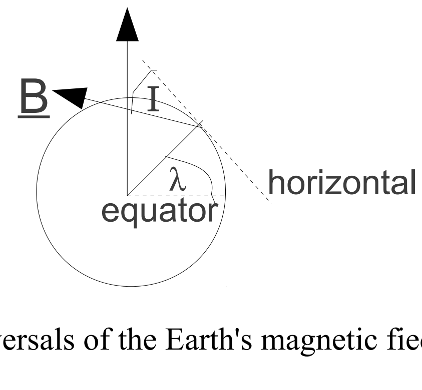
The origin of the magnetic field is the geodynamo, electrical currents in the core give rise to a magnetic field (however the exact mechanism through which this happens is unknown).

Records of the magnetic field can be found in magnetic minerals:

1. Rock forms from melt (e.g. Oceanic basalt).
2. Constituent minerals may be ferro- or ferrimagnetic and cool below their Curie temperature.
3. The magnetisation direction is determined by external geomagnetic field and so records the geomagnetic polarity and inclination/declination.

The *magnetic declination* is the angle between the magnetic and geographical meridians. The *magnetic inclination* is the angle I at which the local field dips below horizontal, depending only on latitude λ for a cylindrically symmetric (dipole) field.

$$\tan(I) = 2\tan(\lambda)$$



Measure I and the latitude that the rock was last magnetised at (i.e. when T went below T_C). This is key evidence for continental drift and plate tectonics.

The pattern of "stripes" found in the crust, parallel to spreading mid ocean ridges suggest there are polarity reversals of the Earth's magnetic fied.

Mid ocean ridges are formed by the upwelling of magma - partially molten upper mantle material feeds into a "magma chamber" which can fissure the crust and flow up through these fissures. N.B.

properties of the asthenosphere relate strongly to its degree of partial melting.

Measuring seismic wave speed "residuals" (i.e. a few percent difference from expected arrival times at many different seismometers), a low velocity zone is seen beneath the ridge axis due to the high temperature of e.g. a magma chamber.

9 - Heat transfer

The "heat budget" at the Earth's surface can be calculated thus:

- Solar input ~ 174 PW = $1.74\times10^{17}\,W$
- $A_{surface}=4\pi R_E^2\approx5\times10^{14}\,m^2$
- Solar power density ~ $350\,Wm^{-2}$

The solar input matches radiative output of the Earth very closely. The heat flow from the Earth's interior is much lower ~ $150\,mW\,m^{-2}$ which gives a total power of ~ $7.5\times10^{13}\,W$
So Solar heating drives the weather, erosion, sedimentation, biosphere.

The internal heat of the Earth has arisen from the K.E. of planetesimals and P.E due to the compression of the Earth during its formation. There is an extra contribution from Radiogenic Heating.

The Earth is considered to be cooling down by transferring heat from the hot core to the cool crust. An estimate from 2007 for the core-mantle boundary temperature gives a result of 3950 K,

From the core to the mantle heat is transferred via an inhomogeneous D" (D-double primed) layer. Through the mantle, convection like processes transfer the heat; from the mantle to the crust/surface mid ocean ridges, hot spots etc. transfer the heat (very inhomogeneous). Note that this is all very hand-wavey.

Seismic tomography data are not consistent with simple convection cells in the mantle. There are two basic models: plume and plate.
The D" layer has widely varying thickness which might affect heat transfer into the lower mantle (there are hot regions, mantle plumes_ It is also thought that subducted oceanic crust can actually reach the CMB.

This "convective" heat transfer may help to drive plate tectonics which are believed to have 3 major forces behind them:

1. Gravitational thrust
2. Mantle drag (some sort of sideways convective motion? Not much evidence)
3. Plate pull force (an already subducted plate can pull the rest of itself down).

10 - Directly Probing the Earth's interior

Can it be done? currently the world's deepest hole is approximately 12 km deep[2]. Are there sensors to withstand the temperature and pressure? Can use diamond or silicon carbide or some similar wide band gap material. Note: there was a proposal in nature in 2003 to reproduce a "mini iron catastrophe" with 10^8-10^{10} kg of liquid Fe to sink a probe through the mantle.

Citations:

1 - Lecture 2 of Dr. Gavin Bell's lecture notes
http://www2.warwick.ac.uk/fac/sci/physics/teach/module_home/px266/notes/ retrieved 21/9/11

2 - http://atlasobscura.com/place/kola-superdeep-borehole retrieved 21/9/11